Effects of voltage fluctuations on the current correlations in mesoscopic Y-shaped conductors

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(Dated: February 22, 2006)

We study current fluctuations in a phase coherent Y-shaped conductor connected to external leads and voltage probes. The voltage probes are taken to have finite impedances and thus can cause voltage fluctuations in the circuit. Applying the Keldysh formulation and a saddle point approximation appropriate for slow fluctuations, we examine at zero temperature the feedback effects on the current fluctuations due to the fluctuating voltages. We consider mesoscopic Y-shaped conductors made of tunnel junctions and of diffusive wires. Unlike two-terminal conductors, we find that for the Y-shaped conductors the current moments in the presence of external impedances cannot be obtained from simple rescaling of the bare moments already in the second moments. As a direct consequence, we find that the cross correlation between the output terminals can become *positive* due to the impedances in the circuit. We provide formulas for the range of parameters that can cause positive cross correlations.

I. INTRODUCTION

Current fluctuations in mesoscopic systems are of fundamental interest theoretically and experimentally since they can reveal information inaccessible from conductance measurements (see, for example, Refs. 1–3). For instance, non-equilibrium noises (or shot noises) can be used to determine the effective charge of quasiparticles in transport; a renowned example is the measurement of fractional charges in quantum Hall systems.[4] On the other hand, the statistics of the charge carriers can be studied in multiterminal setups. The quantity of interest in this case is the cross correlation between different terminals.[1,2] A typical experiment of this type is the solid-state analogue of the Hanbury Brown-Twiss experiment in quantum optics;[5] the electron versions of such experiments have shown elegantly that Fermi statistics suppresses current fluctuations in comparison with uncorrelated charge carriers.[6,7] There have also been proposals for probing the fractional statistics of quasiparticles in quantum Hall systems[8] base on Hanbury Brown-Twiss type setups.[9,10,11]

Typically, in measurements of cross correlations, one injects an incident beam of charge carriers and then splits the beam into two parts using a "beam splitter", such as a Y-shaped conductor.[6,7] By measuring the intensity correlation between the output beams, one can extract information regarding the cross correlations. In most situations, current measurements involve coupling of the sample to external circuits. If the measurement circuit can be idealized as having zero impedance,[7] the voltage across the sample would be non-fluctuating and the current fluctuations are entirely due to intrinsic properties (such as statistics, for example) of the carriers. If, however, the measuring circuit has non-negligible impedance,[6] the voltage across the sample then becomes fluctuating and the current fluctuations in this case will be modified due

to the voltage fluctuations. This is the feedback effect we will be investigating in this paper.

Feedback effects on current moments have previously been considered for two-terminal cases at zero temperature and at finite temperatures.[1,12,13,14,15] The results based on the Langevin formalism concluded that the second moments of current fluctuations can be obtained from the corresponding zero-impedance (intrinsic or "bare") values by simple scaling. However, it was shown, using a Keldysh technique[13,14] and the Langevin formalism,[14,15] that this rescaling breaks down at the third moment. In this work, we study a three-terminal setup (see Fig. 1) using the Keldysh formulation, which complements a previous analysis by the present authors using the Langevin formulation.[16] Although the effects of external impedances on current fluctuations in multiterminal circuits have also been considered by Büttiker and coworkers,[1] they focus mainly on a multiprobe measurement of a two-terminal conductor and thus not directly our geometry here (see, however, Ref. 17 that we shall refer to later).

We will study a mesoscopic, phase coherent Y-shaped conductor (sample A in Fig. 1) connected to a measuring circuit with finite impedances (schematized as Z_a, Z_b, Z_c in Fig. 1). The sample arms of A will be taken to be either tunnel junctions or diffusive wires. In the absence of the external impedances, it is well known that the cross correlation between different terminals, say b and c, is negative due to Fermi statistics of the electrons.[18] With finite impedances in the measuring circuit, however, we will find that voltage fluctuations can modify current fluctuations significantly. In particular, the second moment cannot be obtained from simple rescaling of the corresponding zero-impedance (or "bare") expressions. For instance, the cross correlation will acquire contributions from noise correlators. Since the bare noise correlators are always positive, it is then possible to have
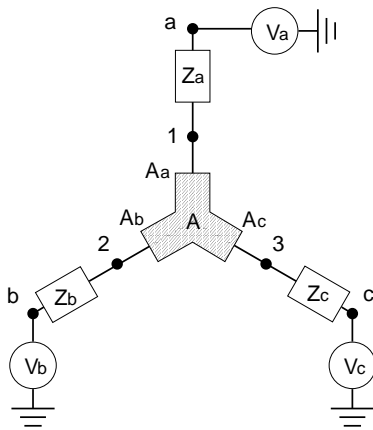
FIG. 1: Schematic for the system considered in this paper. The arms A_a, A_b, A_c of a mesoscopic Y-shaped conductor A are connected to external leads biased, respectively, at voltages V_a, V_b, and V_c. The leads are assumed to have impedances Z_a, Z_b, and Z_c, which are schematized as external resistors connected to the sample arms. The nodes $1, 2, 3$ between the sample arms and the resistors are where voltage fluctuations set in.

positive cross correlations in the appropriate parameter regime.[16]

Theoretical predictions for positive cross correlations in multiterminal setups had previously been made for systems with terminals of non-Fermi liquid ground states,[19] such as superconductors,[20] quantum Hall states,[9,21] Luttinger liquids,[9,22] and ferromagnets.[23,24] In normal systems it has also been predicted for systems with capacitively coupled contacts.[25] Our finding of positive cross correlations due to feedback effects thus provides a new mechanism for positive cross correlation in normal systems. It is of direct experimental relevance since feedback effects are of crucial importance in almost all experiments (cf., for example, Ref. 12). A recent preprint by Rychkov and Büttiker[17] studies the sign change of current cross

correlations in three-terminal conductors due to inelastic scattering, and hence voltage fluctuations. Whereas our voltage fluctuations occur outside our coherent conductor (at nodes 1–3 in Fig. 1), they considered the situation where inelastic scattering occurs inside the Y-shaped conductor where the arms joint together. In our case, as we shall see, the sign of the cross correlation crucially depends on *how* the external impedances are placed: for $V_b = V_c$ (as we shall consider below), we find that Z_b, Z_c tend to make the cross correlation between terminals b and c more positive, while Z_a would make it more *negative* [see Eq. (21) and Appendix C].

We shall derive in Sec. II analytic formulas for the current moments using the Keldysh technique. In Sec. III we will then present numerical results for the cross correlation and discuss in detail its sign change in different parameter regimes. To help focus on the main points, we relegate most details to the Appendices. Finally in Sec. IV we summarize and further discuss our results.

II. FORMULATION

We consider a system schematized as in Fig. 1, where a phase coherent mesoscopic Y-shaped conductor A (the "sample") with arms A_a, A_b, and A_c is connected to a measuring circuit. Each arm A_α ($\alpha = a, b, c$) is connected to an external lead α biased at voltage V_α. The arm A_α is taken to have conductance G_α and the lead connected to it has impedance Z_α. For convenience, we define the dimensionless quantities $g \equiv (h/e^2)(G_a + G_b + G_c)$, $\eta_\alpha \equiv G_\alpha/(G_a + G_b + G_c)$, and $z_\alpha \equiv (e^2/h)Z_\alpha$. Note that it follows from these definitions $\eta_a + \eta_b + \eta_c = 1$. Without lost of generality, we shall take $V_a \geq V_b \geq V_c$ and the downward direction as the positive current direction.

To take voltage fluctuations into account, we apply the Keldysh technique developed in Ref. 13. In this formulation, the current moments are obtained from the generating functional

$$\mathcal{Z}[\Phi, \chi] = \left\langle \overleftarrow{T} \exp\left\{ \frac{i}{e} \int dt \left[\Phi(t) + \frac{1}{2}\chi(t) \right] \hat{I}(t) \right\} \overrightarrow{T} \exp\left\{ \frac{i}{e} \int dt \left[-\Phi(t) + \frac{1}{2}\chi(t) \right] \hat{I}(t) \right\} \right\rangle. \tag{1}$$

Here e is the charge of an electron, $\hat{I}$ is the current operator, $\overleftarrow{T}$ and $\overrightarrow{T}$ are, respectively, the time ordering operators in ascending and descending directions; $\Phi(t) = (e/\hbar) \int_0^t V(t')dt'$ stands for the accumulated phase, and $\chi(t)$ is the counting field. Evaluation of the full expression will then allow one to obtain the *full counting statistics*[26] of transported charges. At the same time, current moments of any order can also be obtained from functional derivatives of $\ln \mathcal{Z}$ with respective $\chi(t)/e$.[13]

To implement the Keldysh approach to current fluc-

tuations in the presence of external impedances in our problem, therefore, our first task is to set up the auxiliary fields Φ and χ, taking into account effects of the external resistors. At the leads $\alpha = a, b, c$, we denote the counting fields as χ_α and the accumulated phases during the detection time τ as $\Phi_\alpha = (e/\hbar) \int_0^\tau V_\alpha dt$. At the nodes $k = 1, 2, 3$, these variables take unknown values χ_k, Φ_k, which have to be integrated over all possible values in the generating functional for the total system. Generalizing the expressions of Ref. 13, one can then obtain the

generating functional for the total system from the convolution of the generating functionals of the leads $(\mathcal{Z}_\alpha)$ and the sample $(\mathcal{Z}_A)$, namely

$$
\mathcal{Z}_{tot}[\Phi_a, \Phi_b, \Phi_c, \chi_a, \chi_b, \chi_c] = \int \prod_{k=1}^{3} \mathcal{D}\Phi_k \, \mathcal{D}\chi_k \, \mathcal{Z}_A[\Phi_1, \Phi_2, \Phi_3, \chi_1, \chi_2, \chi_3] \, \mathcal{Z}_a[(\Phi_a - \Phi_1), (\chi_a - \chi_1)]
$$
$$
\mathcal{Z}_b[(\Phi_b - \Phi_2), (\chi_b - \chi_2)] \, \mathcal{Z}_c[(\Phi_c - \Phi_3), (\chi_c - \chi_3)]. \qquad (2)
$$

Since we are interested in the zero temperature limit, as explained in Ref. 13, for slow enough phase fluctuations (*ie.,* in the low frequency regime) we can approximate the generating functional by defining the action S so that

$$
\mathcal{Z}[\Phi(t), \chi(t)] = \exp\left\{ \int dt S[\dot{\Phi}(t), \chi(t)] \right\}, \qquad (3)
$$

where $\dot{\Phi} = d\Phi/dt$. A saddle point approximation to $\mathcal{Z}_{tot}$ then yields from Eqs. (2) and (3)

$$
\ln \mathcal{Z}_{tot} = \tau S_{tot}(\dot{\Phi}_a, \dot{\Phi}_b, \dot{\Phi}_c, \chi_a, \chi_b, \chi_c)
$$
$$
= \tau S_A(\dot{\Phi}_1, \dot{\Phi}_2, \dot{\Phi}_3, \chi_1, \chi_2, \chi_3) + \tau S_a(\dot{\Phi}_a - \dot{\Phi}_1, \chi_a - \chi_1)
$$
$$
+ \tau S_b(\dot{\Phi}_b - \dot{\Phi}_2, \chi_b - \chi_2) + \tau S_c(\dot{\Phi}_c - \dot{\Phi}_3, \chi_c - \chi_3), \quad (4)
$$

where τ is the detection time and $\dot{\Phi}_k$, χ_k ($k = 1, 2, 3$) are evaluated at their saddle point values. One can thus obtain current moments of any order from derivatives of τS_{tot} with respect to the counting fields χ_α.[13]

To carry out the calculation explicitly, we will need expressions for the actions in the above formula. For the leads, there are exact expressions for the actions (see below). However, in general, this is not the case for the action S_A for the sample. As we will be interested in the second-order current moments in the zero-temperature regime, an expression linear in Φ_α and second order in χ_α will suffice. Therefore, the next step of our calculation is to expand the total action to linear order in Φ_α and quadratic order in χ_α. The coefficients will then yield information for the *renormalized* current moments (that is, modified current moments due to the impedances in the circuit – see below), as we shall now show.

For the leads, the Keldysh actions can be found exactly. The lead α that is connected to the arm A_α via node k has the action

$$
S_\alpha = \frac{i(\chi_\alpha - \chi_k)(\dot{\Phi}_\alpha - \dot{\Phi}_k)}{2\pi z_\alpha}. \qquad (5)
$$

Alternatively, defining $\sigma \equiv i\chi$ and $\phi \equiv \Phi/(2\pi)$, we have

$$
\tau S_\alpha = \frac{(\sigma_\alpha - \sigma_k)(\phi_\alpha - \phi_k)}{z_\alpha}. \qquad (6)
$$

The total action is thus

$$
\tau S_{tot} = \frac{(\sigma_a - \sigma_1)(\phi_a - \phi_1)}{z_a} + \frac{(\sigma_b - \sigma_2)(\phi_b - \phi_2)}{z_b}
$$
$$
+ \frac{(\sigma_c - \sigma_3)(\phi_c - \phi_3)}{z_c} + \tau S_A. \qquad (7)
$$

To linear order in the phase variables ϕ_k, the action for the sample A can be expressed

$$
\tau S_A = \mathcal{S}^{(b)}(\phi_1 - \phi_2) + \mathcal{S}^{(c)}(\phi_1 - \phi_3), \qquad (8)
$$

where $\mathcal{S}^{(\alpha)}$ are functions of σ_1, σ_2, and σ_3. Expansions of the $\mathcal{S}^{(\alpha)}$'s then generate quantities proportional to the current moments.[13] To second order we have

$$
\mathcal{S}^{(\alpha)} \simeq s_b^{(\alpha)}(\sigma_1 - \sigma_2) + s_c^{(\alpha)}(\sigma_1 - \sigma_3) + \frac{s_{bb}^{(\alpha)}}{2!}(\sigma_1 - \sigma_2)^2
$$
$$
+ \frac{s_{cc}^{(\alpha)}}{2!}(\sigma_1 - \sigma_3)^2 + s_{bc}^{(\alpha)}(\sigma_1 - \sigma_2)(\sigma_1 - \sigma_3). \quad (9)
$$

The expansion coefficients are related to the *bare* current moments (*ie.,* current moments in the absence of external resistors – see below). For example, $s_b^{(b)}$ is the dimensionless conductance (*ie.* conductance in units of e^2/h) of arm A_b due to the potential difference between nodes 1 and 2. Likewise, $s_{bb}^{(c)}$ is proportional to the current noise at arm A_b due to the potential difference between nodes 1 and 3. Since there is no any external resistors between each pair of nodes, the moments related to these coefficients are therefore the "bare" ones. These can be calculated in many methods and the results are summarized in Appendix A.

From the saddle point condition

$$
\frac{\partial}{\partial \phi_k}\left(\tau S_{tot}\right) = 0, \qquad (k = 1, 2, 3), \qquad (10)
$$

we get

$$
-\frac{\sigma_a - \sigma_1}{z_a} + \mathcal{S}^{(b)} + \mathcal{S}^{(c)} = 0,
$$
$$
-\frac{\sigma_b - \sigma_2}{z_b} - \mathcal{S}^{(b)} = 0,
$$
$$
-\frac{\sigma_c - \sigma_3}{z_c} - \mathcal{S}^{(c)} = 0. \qquad (11)
$$

Using (11) and (8) in Eq. (7), we find that at the saddle point the total action is reduced to

$$
\tau S_{tot} = \mathcal{S}^{(b)}(\phi_a - \phi_b) + \mathcal{S}^{(c)}(\phi_a - \phi_c). \qquad (12)
$$

Note that we have eliminated all the intermediate variables ϕ_k; $\mathcal{S}^{(b)}$ and $\mathcal{S}^{(c)}$ depend only on the intermediate variables σ_k ($k = 1, 2, 3$). Also, it is interesting to

note that (12) takes the form of τS_A (see Eq. (8)) with $\phi_1 = \phi_a$, $\phi_2 = \phi_b$, and $\phi_3 = \phi_c$. However, this simplicity is deceptive and one should not take it literally. One can check easily that it would otherwise lead to contradictions.

While Eq. (12) is completely general, for simplicity, we shall from now on consider the case of $\phi_b = \phi_c$ by setting $V_a = V$ and $V_b = V_c = 0$. Using the first of the saddle point equations (11), one finds that the total action becomes

$$\tau S_{tot} = \frac{(\sigma_a - \sigma_1)(\phi_a - \phi_b)}{z_a} \,. \tag{13}$$

Note that there is now only one unknown, σ_1, that remains. As the total action contains contributions from the external resistors, an expansion of τS_{tot} with respect to σ_a, σ_b, and σ_c can yield the *renormalized* current moments (*ie.*, current moments in the presence of external impedances) of any orders. For this purpose, in view of Eq. (13), we need only expand σ_1 in terms of σ_a, σ_b, and σ_c. As we shall see, interesting effects arise already in the second moments. Thus, we expand σ_1 to second order and get

$$\frac{(\sigma_a - \sigma_1)}{z_a} \simeq C_b(\sigma_a - \sigma_b) + C_c(\sigma_a - \sigma_c)$$
$$+ \frac{C_{bb}}{2!}(\sigma_a - \sigma_b)^2 + \frac{C_{cc}}{2!}(\sigma_a - \sigma_c)^2$$
$$+ C_{bc}(\sigma_a - \sigma_b)(\sigma_a - \sigma_c) \,. \tag{14}$$

The expansion coefficients here are directly related to the renormalized current moments: $(e^2/h)C_\alpha$ yields the conductance and $(e^3V/h)C_{\alpha\beta}$ the noise correlator/cross correlation. For brevity, we shall later on refer to them simply as the conductance and the noise correlator/cross correlation (and likewise for their bare counterparts).

It is not hard to solve the expansion coefficients in Eq. (14) (see Appendix B). We find the first order coefficients

$$C_b = \frac{1}{z_t}\left[(P+S)s_b^{(b)} + (Q+R)s_b^{(c)}\right] \,,$$
$$C_c = \frac{1}{z_t}\left[(P+S)s_c^{(b)} + (Q+R)s_c^{(c)}\right] \,, \tag{15}$$

and the second order coefficients

$$C_{bb} = \frac{1}{z_t^3}\left\{(P+S)\left[P^2 s_{bb}^{(b)} + Q^2 s_{cc}^{(b)} + 2PQ s_{bc}^{(b)}\right] + (Q+R)\left[P^2 s_{bb}^{(c)} + Q^2 s_{cc}^{(c)} + 2PQ s_{bc}^{(c)}\right]\right\} \,, \tag{16}$$

$$C_{cc} = \frac{1}{z_t^3}\left\{(P+S)\left[S^2 s_{bb}^{(b)} + R^2 s_{cc}^{(b)} + 2SR s_{bc}^{(b)}\right] + (Q+R)\left[S^2 s_{bb}^{(c)} + R^2 s_{cc}^{(c)} + 2SR s_{bc}^{(c)}\right]\right\} \,, \tag{17}$$

$$C_{bc} = \frac{1}{z_t^3}\left\{(P+S)\left[PS s_{bb}^{(b)} + QR s_{cc}^{(b)} + (PR+QS)s_{bc}^{(b)}\right]\right.$$
$$+ \left.(Q+R)\left[PS s_{bb}^{(c)} + QR s_{cc}^{(c)} + (PR+QS)s_{bc}^{(c)}\right]\right\} \,. \tag{18}$$

In these equations

$$z_t \equiv g^2 \eta_a \eta_b \eta_c\left[\left(z_a + \frac{1}{g\eta_a}\right)\left(z_b + \frac{1}{g\eta_b}\right) + \left(z_b + \frac{1}{g\eta_b}\right)\left(z_c + \frac{1}{g\eta_c}\right) + \left(z_c + \frac{1}{g\eta_c}\right)\left(z_a + \frac{1}{g\eta_a}\right)\right]$$
$$= \eta_a(1 + gz_b\eta_b)(1 + gz_c\eta_c) + \eta_b(1 + gz_c\eta_c)(1 + gz_a\eta_a) + \eta_c(1 + gz_a\eta_a)(1 + gz_b\eta_b) \,. \tag{19}$$

is a dimensionless quantity which is proportional to the total resistance of the circuit (though note that the total resistance depends on the arrangement of the bias voltages). The symbols P, Q, R, S stand for

$$P = 1 + z_a\left(s_b^{(c)} + s_c^{(c)}\right) + z_c s_c^{(c)} \,,$$
$$Q = -z_a\left(s_b^{(b)} + s_c^{(b)}\right) - z_c s_c^{(b)} \,,$$
$$R = 1 + z_a\left(s_b^{(b)} + s_c^{(b)}\right) + z_b s_b^{(b)} \,,$$
$$S = -z_a\left(s_b^{(c)} + s_c^{(c)}\right) - z_b s_b^{(c)} \,. \tag{20}$$
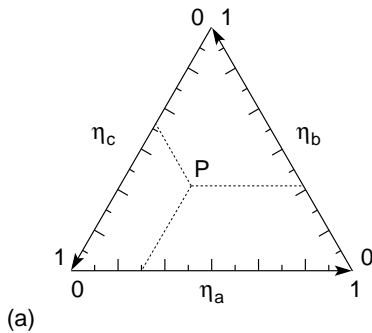
(a)

FIG. 2: (a) Illustration for the triangular coordinate frames adopted for the plots in this paper. For any point P on the plot, the corresponding η_α's can be read off as indicated. The lower panels plot the *bare* cross correlations (namely, $C_{bc} = s_{bc}$ when $z_a = z_b = z_c = 0$) for Y-shaped conductors with arms of (b) tunnel junctions and (c) diffusive wires. In (b) and (c) (and also in other plots in this paper) the unit of C_{bc} is g and the coordinates x, y of the boxed-frame are related to the η_α's by $\eta_a = x$, $\eta_b = (2/\sqrt{3})y$, and $\eta_c = 1 - (2/\sqrt{3})y$; the thick lines over the surfaces depict the contours for $C_{bc} = 0$.

Equations (15)–(20) constitute the main results of this paper; they agree with results obtained from a Langevin formulation approach.[16] The coefficients $s_\alpha^{(\beta)}$, $s_{\alpha\gamma}^{(\beta)}$ here are the same as those of Eq. (9), whose explicit forms are provided in Appendix A. Using elementary circuit theory, one can check easily that C_α ($\alpha = a, b, c$) is exactly the dimensionless conductance (*ie.* conductance in units of e^2/h) of the arm A_α in the presence of external impedances. Also, as one can check from the above expressions, when all external impedances are zero, for example, C_b reduces to the bare moment $(s_b^{(b)} + s_b^{(c)}) = g\eta_a\eta_b$, namely the conductance of arm A_b (note that we are considering $V_b = V_c$ here, thus the conductance is the sum of the two contributions).

The second moments C_{bb}, C_{cc} are the noise correlators and C_{bc} is the cross correlation between arms A_b and A_c (though see above, below Eq. (14)). When all the z_α's are zero, C_{bc} reduces to the bare cross correlation $s_{bc} \equiv s_{bc}^{(b)} + s_{bc}^{(c)}$. As is well known, due to the Fermi statistics of the charge carriers, s_{bc} is always negative.[18] We plot in Fig. 2 the bare cross correlations s_{bc} (C_{bc} for $z_a = z_b = z_c = 0$) for Y-shaped conductors made of tunnel junctions and of diffusive wires. In presenting our results, here and below we shall adopt a triangular coordinate frame (see Fig. 2(a)). This is because for given values of z_α's it is convenient to plot C_{bc} for all possible values of the η_α's using this system of frames. As one can check easily from Fig. 2(a), every point on the triangle satisfies the constraint $\eta_a + \eta_b + \eta_c = 1$.

It should be noticed that, as a consequence of the feedback effects from voltage fluctuations, the second moments are now linear combinations of all bare second moments, instead of simple rescaling of the corresponding bare moments. As demonstrated in Appendix C, the

noise correlators C_{bb}, C_{cc} are always positive, as they should. However, the cross correlation C_{bc} can change sign in different parameter regimes. This is in sharp contrast with the bare cross correlation s_{bc}, which is always negative. As we will see, this occurs because the external impedances induce voltage fluctuations which can revert the sign of the cross correlations. In the next section, we will study this feedback effect in detail. In particular, we will be interested in the cases with the cross correlation C_{bc} turning *positive*.

III.   THE CROSS CORRELATION C_{bc}

In this section, we present our results for the cross correlation C_{bc} in different parameter regimes. The parameter space we are exploring consists of six non-negative parameters $\eta_a, \eta_b, \eta_c, z_a, z_b, z_c$ subjected to the constraint $\eta_a + \eta_b + \eta_c = 1$. As explained in Appendix C, it is useful here to divide the parameter space into two regions

$$\text{region I}: \ z_a\eta_a > z_b\eta_b \text{ and } z_c\eta_c,$$
$$\text{region II}: \ z_a\eta_a < z_b\eta_b, \text{ or } z_c\eta_c, \text{ or both.} \quad (21)$$

One can show that for region I the cross correlation C_{bc} is always negative, while for region II it can flip sign (see Appendix C). For example, if $z_a \neq 0$ while $z_b = z_c = 0$, the cross correlation must be negative no matter what values the conductances η_a, η_b, η_c may be. In the following, we will be interested primarily in region II of the parameter space. Since in experiments the external impedances are usually of the order of the sample resistance,[12] we shall set z_α's of the order of $1/g$ and plot the cross correlation C_{bc} for all values of the η_α's. Typical plots for C_{bc} are shown in Figs. 3 and 4 (see also Figs. 2 and 3 in Ref. 16). One can observe clearly regions of positive cross correlations which grow with increasing external impedances. We shall now provide physical pictures for these results.

Let us examine first the case $z_a = 0$, $z_b = z_c \equiv z \neq 0$. This corresponds to the situation when one attempts to measure the cross correlation C_{bc} by connecting the arms A_b and A_c to voltage probes, while leaving A_a unmeasured. As is obvious from Eq. (21), in this case we are entirely in region II of the parameter space since $z_a\eta_a = 0$ always. It is therefore possible to have large areas of positive cross correlations in the η-triangle, as discovered in Ref. 16 (see Figs. 2 and 3 there). For instance, for $z = 10g^{-1}$, it was found that the cross correlation is almost always positive. In fact, the large z behavior of C_{bc} is determined by the coefficient of its leading term. It is not hard to check that for $z_a = 0$ while $z_b = z_c \neq 0$, the leading coefficient of C_{bc} is *always* positive. Therefore, for z large enough, C_{bc} can be positive over the whole η-triangle (*ie.*, whatever values of the η_α's)!

To understand the above results, we note that an interesting feature in the plots for C_{bc} (see Figs. 2 and 3 in Ref. 16 and also Figs. 3 and 4 above) is that positive
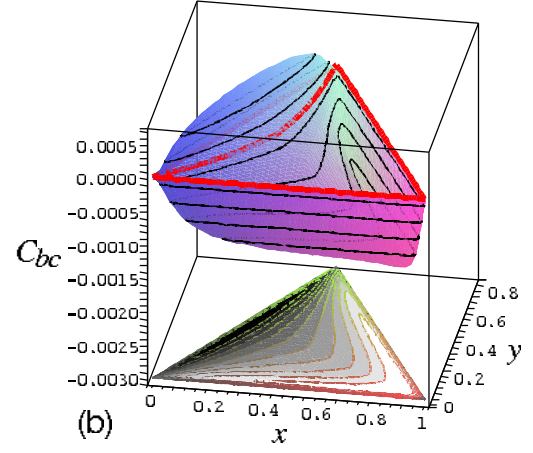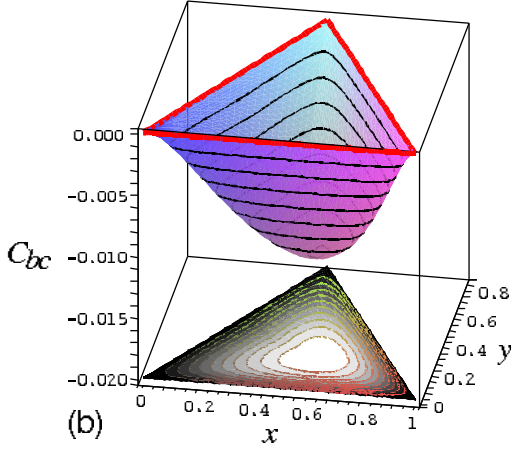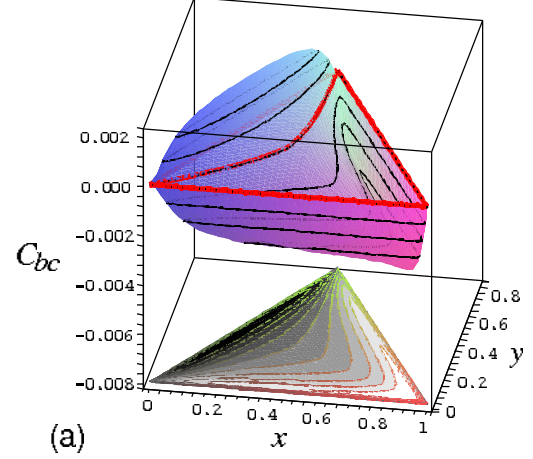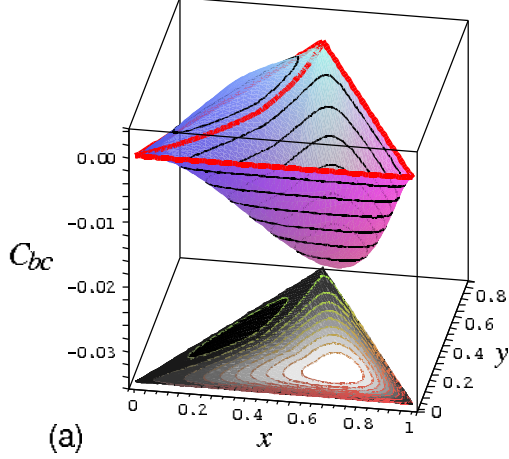
FIG. 3: Plots for the cross correlations for Y-shaped conductors with arms of (a) tunnel junctions and (b) diffusive wires where the external impedances are $z_a = z_b = z_c = 1/g$.

FIG. 4: Plots for the cross correlations of Y-shaped conductors with (a) tunnel junctions and (b) diffusive wires in the arms. Here the external impedances are $z_a = z_b = z_c = 10g^{-1}$. Note the different vertical scales in these two plots.

cross correlations develop more easily for small η_a and large z_b, z_c. This can be understood physically as follows (for more quantitative analysis, see later). When there is a positive fluctuation of the current through, say the arm A_b, there is a corresponding increase in the potential at node 2 in Fig. 1. This voltage fluctuation in turn will lead to an extra current through the arm A_c, thus giving a *positive* contribution to the cross correlation C_{bc}. This contribution will in particular be large for small η_a, since most of this fluctuating current will flow through c. We have a net positive C_{bc} if this contribution overwhelms the "bare" negative correlation contribution given by s_{bc}. Therefore, the positive region starts near small η_a, and grows with increasing z_b and z_c.

More quantitatively, let us consider the case with $\eta_b = \eta_c$. Recalling that $z_a = 0$ and $z_b = z_c \equiv z$ here, one gets from Eq. (20) for small η that $P = R = 1 + \zeta$ and $Q = S = \zeta$ with $\zeta \equiv g z \eta_b^2$. From (18) we then find

$C_{bc} \propto (1 + 2\zeta) [\zeta(1 + \zeta)(s_{bb} + s_{cc} + 2s_{bc}) + s_{bc}]$. Note that $(s_{bb} + s_{cc} + 2s_{bc})$ is proportional to the bare shot noise for the total current and hence positive definite. Thus for sufficiently large z (hence ζ), one can have $C_{bc} > 0$. Note also that for tunnel junctions s_{bc} is proportional to η_a^2, whereas for diffusive wires it is proportional to η_a (see Appendix A). This explains why it is easier for tunnel junctions to get positive cross correlations (see Figs. 2 and 3 in Ref. 16 and also Figs. 3 and 4 above).

Let us now turn to another case which may also have experimental interest. Suppose one connects the sample to three voltage probes with (almost) identical impedances, the circuit then corresponds to $z_a = z_b = z_c \equiv z$ in our calculation. Our results for $z = 1/g$ are shown in Fig. 3. Similar to the previous case, for tunnel junctions, there is an appreciable region of positive cross correlation, while none for the diffusive wires (cf. Fig. 2 in

Ref. 16). According to our analysis above, for larger values of z the region for positive cross correlations should grow. Figure 4 shows the results for $z = 10g^{-1}$; the trend is clearly in accordance with what we concluded above. We remark that the region for positive cross correlations is here bounded by the lines $\eta_a < \eta_b$ and $\eta_a < \eta_c$. As discussed in Appendix C, for $z_a = z_b = z_c$ one can show that whenever η_a is greater than η_b or η_c, one must have $C_{bc} < 0$. Note that this restricts further the region for positive cross correlations [cf., region II of (21)].

Finally, we consider the simple case $z_a = z_c = 0$ while $z_b \neq 0$. Though may not of practical interest, this case may help further understand our results. In the large z_b limit, one can check that the leading coefficient of C_{bc} would be positive if $\eta_a < \eta_c$. Physically, this is because the resistor connected to the arm A_b causes voltage fluctuations which act back on the current. The reversed current would go into arm A_a if the conductance of arm A_a is greater than that of A_c (ie., $\eta_a > \eta_c$). However, if $\eta_a < \eta_c$ the fluctuating current would go into arm A_c, leading to enhancement of the current I_c. This is the reason for the positive cross correlation C_{bc} between arms A_b and A_c.

IV. DISCUSSIONS AND CONCLUSIONS

In summary, we have applied a Keldysh technique to study effects of voltage fluctuations on the current correlations in a mesoscopic, phase coherent Y-shaped conductor that is connected to a measuring circuit with finite impedance. We find that, at zero temperature and at low frequencies, the current moments are significantly modified by the feedbacks from voltage fluctuations due to the finite impedance in the circuit. In particular, the current moments cannot be obtained from simple rescaling of the bare moments already in the second moments. An interesting consequence of this is that there can be positive cross correlations in appropriate parameter regimes.

Positive cross correlations are usually associated with "bunching" behavior of charge carriers, which would also imply enhanced Fano factors. However, it is also known that positive cross correlation is not a *sufficient* condition for electron bunching. Indeed there are existing examples of positive cross correlations that do not go with electron bunching.[24] We have calculated the Fano factors for the total current in our circuit and find that there is in fact no any electron bunching here – even when the cross correlations are positive. For example, for $z_a = z_b = z_c = 1/g$ we plot in Fig. 5 the Fano factors[27] F for the total current for tunnel junctions and diffusive wires normalized to their bare values. It is seen that the Fano factors at most recover their bare values in the regions with positive cross correlations and never go beyond them. Thus there is no any trace of electron bunching here. The positive cross correlation we have found is therefore, instead of electron bunching, purely a feedback effect due to voltage fluctuations. Our result thus shows that one has to
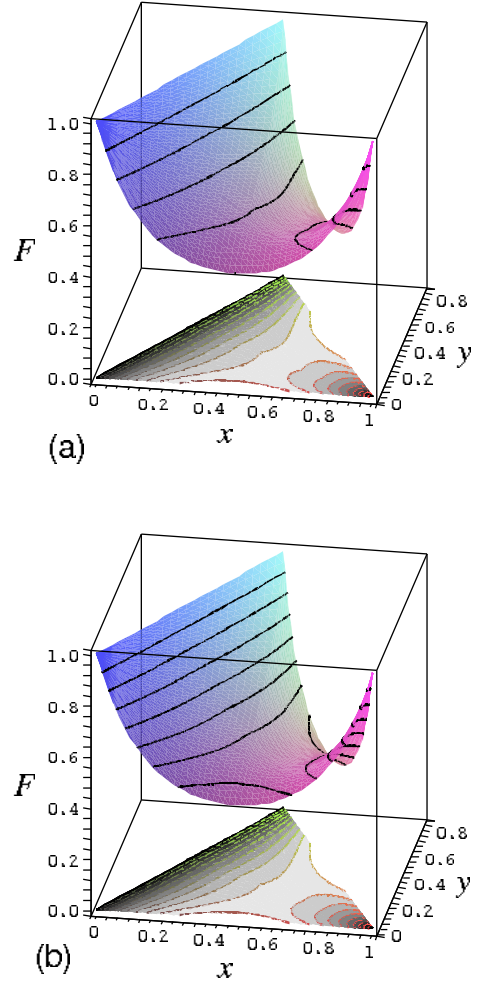

FIG. 5: The Fano factors for Y-shaped conductors with (a) tunnel junctions and (b) diffusive wires in the arms. Here the impedances are $z_a = z_b = z_c = 1/g$ and we have normalized the Fano factors to their bare values.

be very careful in interpreting experimental data that exhibit positive cross correlations, especially when the measuring circuit is expected to have a finite impedance.

APPENDIX A: BARE CURRENT MOMENTS AND THE s-COEFFICIENTS

In the Keldysh technique, current moments of any order can be obtained from derivatives of the generation functional with respect to the auxiliary field σ.[13,14] Therefore, to obtain current moments up to second order, we expand the action for the sample to quadratic order

$$
\begin{aligned}
\tau S_A = {} & \langle \hat{q}_b \rangle (\sigma_1 - \sigma_2) + \langle \hat{q}_c \rangle (\sigma_1 - \sigma_3) + \\
& \langle\langle \hat{q}_b^2 \rangle\rangle (\sigma_1 - \sigma_2)^2 / 2! + \langle\langle \hat{q}_c^2 \rangle\rangle (\sigma_1 - \sigma_3)^2 / 2! + \\
& \langle\langle \hat{q}_b \hat{q}_c \rangle\rangle (\sigma_1 - \sigma_2)(\sigma_1 - \sigma_3) \,.
\end{aligned} \tag{A1}
$$

Here $\hat{q}_\alpha = (1/e) \int_0^\tau \hat{I}_\alpha dt$ is the operator for transferred charges through arm A_α during the time interval τ; $\langle \cdots \rangle$ is a quantum average over ensembles and $\langle\langle \cdots \rangle\rangle$ is the irreducible (or cumulant) average: $\langle\langle \hat{P}\hat{Q} \rangle\rangle \equiv \langle \hat{P}\hat{Q} \rangle - \langle \hat{P} \rangle \langle \hat{Q} \rangle$.

As illustrated in Fig. 1, the arms of sample A are connected to nodes with (intermediate) phase variables ϕ_1, ϕ_2, ϕ_3. Since all external impedances are beyond this region, the related moments $\langle \hat{q}_\alpha \rangle$, $\langle\langle \hat{q}_\alpha \hat{q}_\beta \rangle\rangle$ are thus simply the *bare* ones. These moments of transferred charges are related to the s-coefficients (which are proportional to the *bare* current moments) defined in the text. Comparing the above equation with Eqs. (8) and (9), we find the first moments

$$
\langle \hat{q}_\alpha \rangle = s_\alpha^{(b)}(\phi_1 - \phi_2) + s_\alpha^{(c)}(\phi_1 - \phi_3) \tag{A2}
$$

and the second moments

$$
\langle\langle \hat{q}_\alpha \hat{q}_\beta \rangle\rangle = s_{\alpha\beta}^{(b)}(\phi_1 - \phi_2) + s_{\alpha\beta}^{(c)}(\phi_1 - \phi_3) \,. \tag{A3}
$$

The s-coefficients can be calculated using the technique developed by Nazarov and coworkers[28] and also in other methods.[29] To avoid distractions from the main points, we shall not present these calculations.[30] We shall simply list the results and point out the essential properties (such as their sign characteristics) which will be crucial to our calculation.

1. Tunnel junctions

When the arms of sample A are made of tunnel junctions, we find the first moments yield

$$
\begin{aligned}
s_b^{(b)} &= g\eta_b(\eta_a + \eta_c), & s_b^{(c)} &= -g\eta_b\eta_c, \\
s_c^{(b)} &= -g\eta_b\eta_c, & s_c^{(c)} &= g\eta_c(\eta_a + \eta_b) \,.
\end{aligned} \tag{A4}
$$

The second moments take different forms depending on the relative magnitudes of the intermediate phase vari-

ables ϕ_1, ϕ_2, and ϕ_3. For $\phi_1 > \phi_2 > \phi_3$, one can find

$$
\begin{aligned}
s_{bb}^{(b)} &= g\eta_b[\eta_a(1 - 2\eta_a\eta_b) - \eta_c(1 - 2\eta_b\eta_c)] \,, \\
s_{bb}^{(c)} &= g\eta_b\eta_c(1 - 2\eta_b\eta_c) \,, \\
s_{cc}^{(b)} &= -g\eta_b\eta_c(1 - 2\eta_a\eta_c - 2\eta_b\eta_c) \,, \\
s_{cc}^{(c)} &= g\eta_c[\eta_a(1 - 2\eta_a\eta_c) + \eta_b(1 - 2\eta_a\eta_c - 2\eta_b\eta_c)] \,, \\
s_{bc}^{(b)} &= g\eta_b\eta_c(1 - 2\eta_a^2 - 2\eta_a\eta_c - 2\eta_b\eta_c) \,, \\
s_{bc}^{(c)} &= -g\eta_b\eta_c(1 - 2\eta_a\eta_c - 2\eta_b\eta_c) \,.
\end{aligned} \tag{A5}
$$

For $\phi_1 > \phi_3 > \phi_2$,[31] base on simple symmetry considerations, one can obtain the expressions for the s-coefficients by exchanging the indices b, c in the formulas above. For example, $s_{cc}^{(b)}$ can be obtained from the above expression for $s_{bb}^{(c)}$ with all indices of its right hand members making the exchange $b \leftrightarrow c$.

From these expressions for the s-coefficients, it is not difficult to show that when $\phi_1 > \phi_2 > \phi_3$ the second order coefficients satisfy

$$
s_{bb}^{(c)} > 0, \quad s_{cc}^{(c)} > 0, \quad s_{cc}^{(b)} < 0, \quad s_{bc}^{(c)} < 0 \tag{A6}
$$

always, while $s_{bb}^{(b)}$ and $s_{bc}^{(b)}$ do not have definite signs for general values of the η_α's. For example, using $\eta_a + \eta_b + \eta_c = 1$, one can write from (A5)

$$
s_{bb}^{(c)} = g\eta_b\eta_c[(\eta_a + \eta_b + \eta_c)^2 - 2\eta_b\eta_c] \,, \tag{A7}
$$

which is obviously always positive since all η_α's are nonnegative. Similarly one can check for the other coefficients. We shall make use of these properties in the following when determining the signs of the renormalized (current) moments (the C-coefficients). In the case of $\phi_1 > \phi_3 > \phi_2$, as pointed out above, everything follows from exchanging b and c. Therefore, it is now the $s_{\alpha\beta}^{(b)}$ terms that have definite sign, while not the $s_{\alpha\beta}^{(c)}$ terms (except $s_{bb}^{(c)}$, though).

In our calculation we also find it convenient to introduce the "combined" s-coefficients

$$
s_\alpha \equiv s_\alpha^{(b)} + s_\alpha^{(c)} \quad \text{and} \quad s_{\alpha\beta} \equiv s_{\alpha\beta}^{(b)} + s_{\alpha\beta}^{(c)} \,. \tag{A8}
$$

Therefore, we have

$$
\begin{aligned}
s_b &= g\eta_a\eta_b, \quad s_c = g\eta_a\eta_c, \quad s_{bc} = -2g\eta_a^2\eta_b\eta_c, \\
s_{bb} &= g\eta_a\eta_b(1 - 2\eta_a\eta_b), \quad s_{cc} = g\eta_a\eta_c(1 - 2\eta_a\eta_c) \,.
\end{aligned} \tag{A9}
$$

It is easy to check that for the second order terms

$$
s_{bb} > 0, \quad s_{cc} > 0, \quad s_{bc} < 0 \,. \tag{A10}
$$

These are indeed what one would have expected, in view of the fact that these s-coefficients are exactly the *bare* current moments (namely the current moments at zero impedance – see text): s_{bb}, s_{cc} are the *bare* noise correlators and s_{bc} the *bare* cross correlations, which must take the signs above. Note that Eqs. (A9) and (A10) are valid for both possible arrangements of the intermediate phase variables.

2. Diffusive wires

When the sample arms are made of diffusive wires, calculation shows that the first order coefficients are the same as those for tunnel junctions, namely Eq. (A4). In the case of $\phi_1 > \phi_2 > \phi_3$, we find the second order coefficients

$$s_{bb}^{(b)} = \frac{g}{3}\eta_b(\eta_a - \eta_c)\,, \qquad s_{bb}^{(c)} = \frac{g}{3}\eta_b\eta_c(1 + 2\eta_a)\,,$$

$$s_{cc}^{(b)} = \frac{g}{3}\eta_b\eta_c(2\eta_a - 1)\,, \qquad s_{cc}^{(c)} = \frac{g}{3}\eta_c(\eta_a + \eta_b)\,,$$

$$s_{bc}^{(b)} = \frac{g}{3}\eta_b\eta_c(1 - 2\eta_a)\,, \qquad s_{bc}^{(c)} = -\frac{g}{3}\eta_b\eta_c\,. \qquad \text{(A11)}$$

As before, for $\phi_1 > \phi_3 > \phi_2$ one can obtain the expressions for the s-coefficients by exchanging b and c in the formulas above.

Just like for tunnel junctions, here we also define the "combined" s-coefficients as in Eq. (A8). Thus, we have s_b, s_c the same as in Eq. (A9) and

$$s_{bb} = \frac{g}{3}\eta_a\eta_b(1 + 2\eta_c)\,, \qquad s_{cc} = \frac{g}{3}\eta_a\eta_c(1 + 2\eta_b)\,,$$

$$s_{bc} = -\frac{2g}{3}\eta_a\eta_b\eta_c\,. \qquad \text{(A12)}$$

As one can check easily, the s-coefficients for diffusive wires also have the same sign characteristics as those for tunnel junctions. In other words, Eqs. (A6) and (A10) are valid here as well.

APPENDIX B: ALGEBRA FOR SOLVING THE C-COEFFICIENTS

In this Appendix, we summarize briefly the way to solve for the C-coefficients.

Although in calculating the renormalized current moments one shall need only the expansion of σ_1 (see Eq. (13)), to find the expansion coefficients, however, one has to solve the coupled saddle point equations (11). Therefore, we expand all the intermediate variables σ_k, $k = 1, 2, 3$ as the following

$$\frac{(\sigma_{\alpha_k} - \sigma_k)}{z_{\alpha_k}} \simeq C_b^{(k)}(\sigma_a - \sigma_b) + C_c^{(k)}(\sigma_a - \sigma_c)$$
$$+ \frac{C_{bb}^{(k)}}{2!}(\sigma_a - \sigma_b)^2 + \frac{C_{cc}^{(k)}}{2!}(\sigma_a - \sigma_c)^2$$
$$+ C_{bc}^{(k)}(\sigma_a - \sigma_b)(\sigma_a - \sigma_c)\,, \qquad \text{(B1)}$$

where the subscript

$$\alpha_k = \begin{cases} a & \text{for } k = 1, \\ b & \text{for } k = 2, \\ c & \text{for } k = 3. \end{cases} \qquad \text{(B2)}$$

Substituting the above expansions into the saddle point equations (11) and comparing coefficients, we get the simultaneous equations for the C-coefficients, which can then be solved easily. Since we shall need only the $C^{(1)}$ coefficients for our calculation, the superscripts are omitted in the text.

APPENDIX C: SIGN CHARACTERISTICS OF THE RENORMALIZED SECOND ORDER MOMENTS

In this Appendix we examine the sign characteristics of the renormalized second order moments (the second order C-coefficients). We will show that the noise correlators C_{bb}, C_{cc} are always positive, while the cross correlation C_{bc} can change sign in different parameter regimes.

We study first the noise correlator C_{bb} and consider the case $\phi_1 > \phi_2 > \phi_3$. As noted above, in this case s_{bb}, s_{cc} and $s_{bb}^{(c)}, s_{cc}^{(c)}$ are always positive, while $s_{bc}, s_{bc}^{(c)}$ are always negative (see Eqs. (A6), (A10)). To determine the sign of C_{bb}, it is therefore preferable to eliminate the $s^{(b)}$ coefficients in favor of the $s^{(c)}$'s. Noting that

$$(Q + R) = (P + S) + g\eta_a(z_b\eta_b - z_c\eta_c)\,, \qquad \text{(C1)}$$

one can rewrite the terms in the braces in Eq. (16) as

$$\Gamma_{bb} \equiv (P + S)\left[P^2 s_{bb} + Q^2 s_{cc} + 2PQ s_{bc}\right]$$
$$+ g\eta_a(z_b\eta_b - z_c\eta_c)$$
$$\times \left[P^2 s_{bb}^{(c)} + Q^2 s_{cc}^{(c)} + 2PQ s_{bc}^{(c)}\right]. \qquad \text{(C2)}$$

From the explicit forms of the s-coefficients, one can show that the following inequalities hold

$$(s_{bb})(s_{cc}) \geq (s_{bc})^2\,, \quad (s_{bb}^{(c)})(s_{cc}^{(c)}) \geq (s_{bc}^{(c)})^2\,. \qquad \text{(C3)}$$

Therefore, one has

$$(P^2 s_{bb} + Q^2 s_{cc} + 2PQ s_{bc})$$
$$\geq 2|PQ|\sqrt{(s_{bb})(s_{cc})} + 2PQ s_{bc}$$
$$\geq 2|PQ||s_{bc}| + 2PQ s_{bc} \geq 0\,. \qquad \text{(C4)}$$

Similarly, one can show that $(P^2 s_{bb}^{(c)} + Q^2 s_{cc}^{(c)} + 2PQ s_{bc}^{(c)}) \geq 0$ also. For the prefactors in front of the two terms: from the explicit forms of the s-coefficients, obviously $(P + S) \geq 0$; moreover, it is easy to show that for $\phi_2 > \phi_3$, one must have $z_b\eta_b \geq z_c\eta_c$. Therefore, we conclude that the quantity in Eq. (C2) is always positive and hence C_{bb}.

For the case with $\phi_3 > \phi_2$, the proof proceeds very similarly. However, in this case, it is the coefficients $s_{bb}^{(b)}, s_{cc}^{(b)}$ that are always positive and $s_{bc}^{(b)}$ always negative. It is therefore helpful to express $(P + S)$ in terms of $(Q + R)$, eliminating the $s^{(c)}$'s in favor of the $s^{(b)}$'s. Using

$$(P + S) = (Q + R) + g\eta_a(z_c\eta_c - z_b\eta_b) \qquad \text{(C5)}$$

one can then proceed as before. Noting that $(Q + R) > 0$ always and that when $\phi_3 > \phi_2$ one must have $z_c\eta_c > z_b\eta_b$, one can show easily that C_{bb} is again positive definite. Similar calculations as above (with P, Q replaced by S, R) can also show that C_{cc} is always positive. Indeed, these results can be anticipated as C_{bb} and C_{cc} are both autocorrelators and hence must be always non-negative.

We now turn to the cross correlation C_{bc} and demonstrate the calculation for the case $\phi_1 > \phi_2 > \phi_3$; the calculation for $\phi_1 > \phi_3 > \phi_2$ proceeds very similarly. As for C_{bb}, we use Eq. (C1) and rewrite the braced terms in Eq. (18) as

$$\begin{aligned} \Gamma_{bc} &\equiv (P+S)\left[PSs_{bb} + QRs_{cc} + (PR+QS)s_{bc}\right] \\ &+ g\eta_a(z_b\eta_b - z_c\eta_c) \\ &\times \left[PSs_{bb}^{(c)} + QRs_{cc}^{(c)} + (PR+QS)s_{bc}^{(c)}\right] . \quad \text{(C6)} \end{aligned}$$

Note again that $z_b\eta_b > z_c\eta_c$ here (as $\phi_2 > \phi_3$) and that $(P+S)$, s_{bb}, s_{cc}, $s_{bb}^{(c)}$, and $s_{cc}^{(c)}$ are always positive, while s_{bc} and $s_{bc}^{(c)}$ are always negative. Applying the explicit forms of Q and S, one can show easily that once $z_a\eta_a$ is greater than both $z_b\eta_b$ and $z_c\eta_c$, the quantity Γ_{bc} will be negative definite, hence follows the negative cross correlation. However, if $z_a\eta_a$ is less than $z_b\eta_b$, or $z_c\eta_c$, or both,

Γ_{bc} can take negative or positive values, and thus *positive* cross correlations. This motivates us to divide the parameter space (consisting of $\eta_a, \eta_b, \eta_c, z_a, z_b, z_c$) into two parts according to the sign characteristics of C_{bc}: in one part (region I) it is always negative, while in the other (region II) it can change sign (see Eq. (21)).

For the special case with $z_a = z_b = z_c \equiv z$, we note that we can further limit the region of positive cross correlations to the area with η_a less than *both* η_b and η_c. This can be proved straightforwardly (though the algebra is tedious) using the explicit forms of C_{bc} and the s-coefficients. By expressing C_{bc} as a polynomial of z and checking the signs of the coefficients order by order, one can show that C_{bc} must be negative provided $\eta_a > \min\{\eta_b, \eta_c\}$. Thus, together with the general criterion above, one obtains the limits for positive cross correlations here to be η_a less than *both* η_b and η_c.

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